Gearbox Design for Electric Bicycle

B ME 342

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Problem Statement and Design Constraints

We were tasked to design a mid-motor for a direct drive electric bike for eBikes-R-Us. They requested that we design a double reduction gearbox to accommodate their previously designed motor. The gearbox needs to connect to the bicycle crank, and fit with the following specifications:

- Motor produces 350 W of power at 2000 rpm
- Double reduction gearbox converts to output of 80 rpm
- Width of gearbox may not exceed 125 mm
- Motor body is 90 mm by 60 mm
- Accommodate torque of rider pedaling without failure
- Minimize size
- Minimize noise
- Minimize weight

Design Approach and Necessary Assumptions

Design Approach

When we began our calculations, we created an excel document which complied necessary equations, and related variables to one another so that if we changed anything, we didn't have to recompute our numbers. We sketched an initial design consisting of three shafts: one connecting to the motor, one connecting to the pedals, and one intermediate shaft holding gears 2 and 3. We started by calculating the gearing ratio to be 1:25 and taking the square root to find the gearing ratio for each set of teeth. From here we selected a module through trial and error. As we changed the module, we checked how the diameter of the gears changed, making sure we had enough clearance to accommodate the motor. We wanted to make sure we had at least 45 mm of clearance between the midpoint of shaft A and the top of gear 3 to fit the motor safely in our design. This constraint helped us select a module of 1.75 mm/teeth to give us our desired diameters. From here we performed many calculations while keeping in mind the following assumptions:

Necessary Assumptions

Main Assumptions for Design

- Factor of safety is 1.2
  - Our parts are contained inside a secured housing and have relatively small loads applied to them.
  - This security allows us to use a lower factor of safety to reduce cost, weight, and size of our parts.
- Gearbox is mounted so that the transmitted load from the gear, $W_{t,4}$, is parallel to the weight of the rider.
  - This maximizes the reaction forces on the bearings, providing an extra layer of safety for the rider.

Assumptions for Gear Design

- Ideal Teeth
  - "Teeth are perfectly formed, smooth, and absolutely rigid" (Shigley page 657).
    - Without this assumption calculations are unrealistic to perform because application forces will cause deflections (Shigley page 657).
- Full Teeth
  - k = 1, Addendum = 1/Diametral Pitch, Dedendum = 1.25/Diametral Pitch
  - Full teeth are more common, and less expensive to manufacture.
  - The size saving benefits of stub teeth has a minimal effect to the overall size of our gearbox, so we have opted to prioritize reducing costs.
- Pressure Angle (Φ) = 20 degrees
  - This is a standard pressure angle for gears.
    - This is beneficial because it means that it is a well-researched set up, and manufacturing infrastructure already exists.
- Use preferred modules for gears
  - This also reduces manufacturing costs because it means that you don't have to make specialized parts for this product.

- Use nylon as the material for gears
  - Nylon reduces the noise of the gear train.
    - Noise control is important to our customer, and noise reduction allows them to increase profits.
- $\gamma = 0$
  - Not using bevel gears.
  - For straight angle spur gears, gamma is 0.
- Spur Gears
  - Spur gears are less complex and are therefore easier for the consumer to maintain.
  - They also allow us to use nylon for our gear material, reducing the noise and increasing the profits made off the bike.
- Transmitted load uniformly distributed across the face.
  - This is a necessary assumption to make to use the Lewis equation, otherwise the gear teeth would be too complex to analyze.
- Dynamic factor (K_v) is equal to 1, unity
  - We are assuming our gears will be in motion, so we need to take this into account in order to ensure that they will not fracture under normal use.
  - However, we are using Nylon gears we cannot use any of the equations for the dynamic factor in Shigley because they only apply to metal gears. Because there isn't a lot of literature about the dynamic factor on nylon gears we were told we can assume unity, which means we can set K_v equal to 1.

Assumptions for Bearing Selection

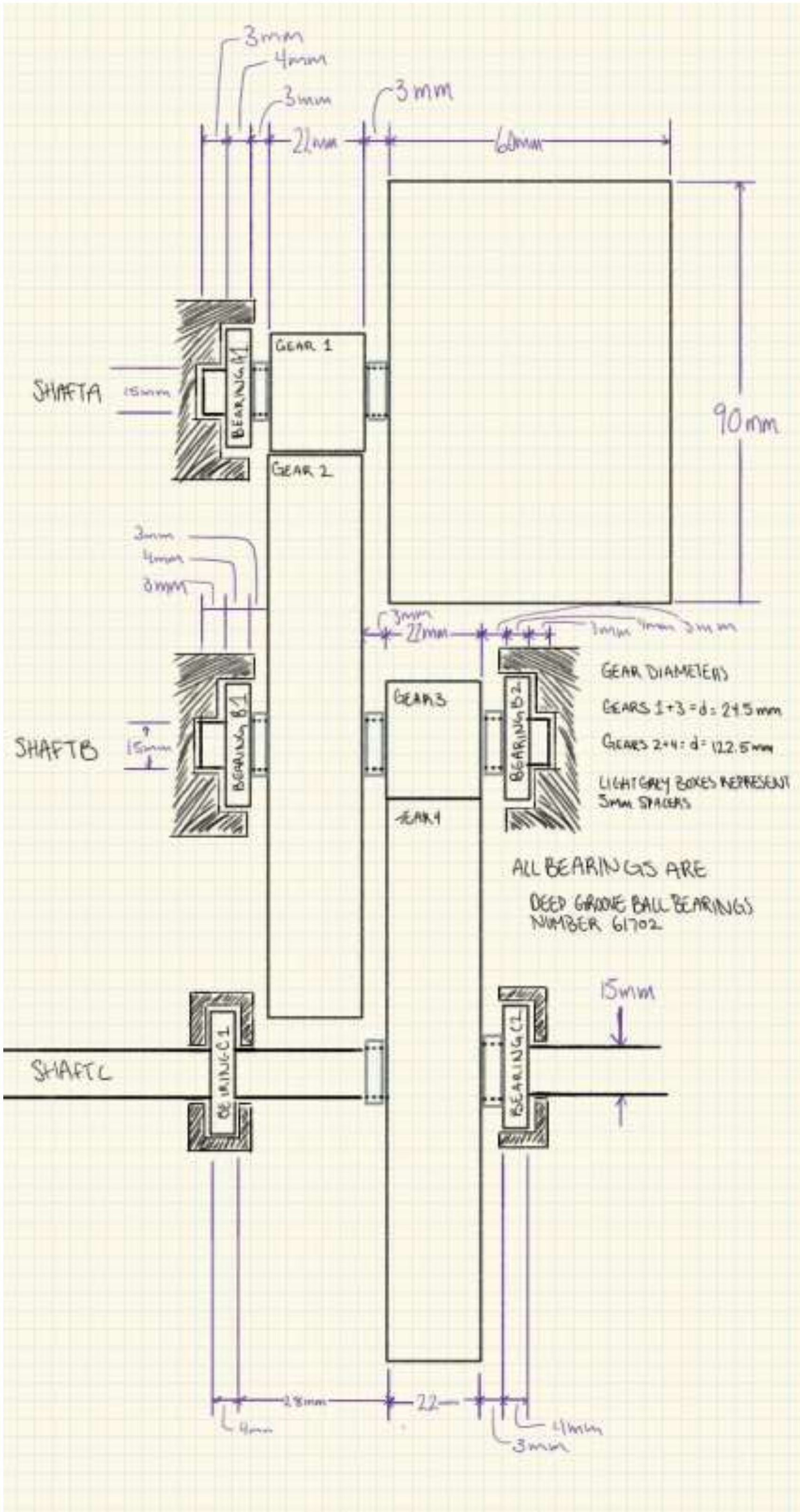
- $a_2 = a_3 = 1$
  - We made this assumption because this is an initial design, so we assumed ideal conditions for the bearing such as:
    - It will be kept clean (a_3)
    - It will be well lubricated (a_3)
    - We are not using a special bearing (a_2)
    - The bearing was heat treated and finished properly (a_2)

- Our bearing life calculation is based on the Weibull distribution
  - This is the standard that Timken uses.
- Reliability is 90%, yielding an a_1 value of 0.64
  - We have two bearings to we needed to split the 90% reliability between the two by taking the square root of 90%, which is 95%.
- Our e-value is 3
  - This is the standard the Timken manual uses for ball bearings.
- L10 is 150,000 cycles
  - This is consistent with the expected lifetime of a standard bike and should give our electric bike a satisfactorily long life before failure occurs.
- Housing is strong enough to support loads from bearings and other sources
  - We were told this by our customer.

Assumptions for Shaft Design

- Drive shaft is 150 mm
  - This is a standard length for a bicycle drive shaft based on market research.
- Crank length is 170 mm
  - This is a standard length for bicycle cranks attached to the pedals based on market research.
- Mass of rider is 120 kg
  - The average person weighs roughly 90 kg.
  - We want to overestimate the mass of the rider, so we added 30 kg to our rider calculations.
    - While this doesn’t match with our factor of safety of 1.2, there is more variability in the weight of people, so we chose a mass larger than our factor of safety dictated.
- Entire mass of rider is placed on pedals, distributed evenly between pedals
  - This is a worst case scenario, so we prepared our calculations with this in mind to provide the rider with maximum safety.
  - This allows us to calculate the maximum torque acting on the shaft and the pedals using this principle.

Gearbox Assembly Diagram



Calculations

Gearing Ratio

Determining the necessary gearing ratio was a simple matter of comparing the input and output shaft speeds, as shown below:

$$\frac{n_{in}}{n_{out}} = \frac{2000\,rpm}{80\,rpm} = 25$$

Since Shigley (p.679) recommends using more than a single pair of gears for gearing ratios greater the 1:10, we elected to split our gear train into a set of two 1:5 pinion/gear pairs.

Module & Teeth numbers

We started by selecting a module of 1.5 mm/teeth for our gears and using equation 13-11 to calculate the minimum number of teeth necessary on the pinion to prevent interference. After using the resulting value and our initial module selection to determine the pitch diameters of the gears (using equation 13-2), we realized that shaft B was colliding with the motor. We therefore updated our module to 1.75 mm/teeth and recalculated our minimum number of teeth, as shown below.

$$N_p = \frac{2k}{(1+2m)sin^2(\varphi)}\left(m + \sqrt{m^2 + (1+2m)sin^2(\varphi)}\right) \quad (eq. 13-11)$$

$$N_p = \frac{2(1)\left(1.75 + \sqrt{(1.75)^2 + (1+2(1.75))sin^2(20°)}\right)}{(1+2(1.75)sin^2(20°)} = 13.85\,teeth \rightarrow 14\,teeth$$

We then found N_G by multiplying by the gearing ratio:

$$N_G = N_p * Gearing\ Ratio = 14\,teeth * 5 = 70\,teeth$$

Diameter & Pitch

We found the diameters and pitch of our gears using equations 13-2 and 13-1, respectively:

$$m = \frac{d}{N} \rightarrow d = mN \quad (eq. 13-2)$$

$$d_G = (1.75)(70) = 122.5\ mm \mid d_p = (1.75)(14) = 24.5\ mm$$

$$P = \frac{N}{d} = \frac{1}{m} = \frac{1}{1.75} = 0.5714 \frac{teeth}{mm} \quad (eq. 13 - 1)$$

Finally, we collect all of our calculations into Table 1.

Table 1: Gear Tooth and Angular Velocity Calculations

Property	Value	Units	Source
ω_A	2000	rpm	Given
ω_B	400	rpm	Chapter 18
ω_C	80	rpm	Given
e	0.04		EQ 13-31
Gearing Ratio	25		Ng/Np
module	1.75	Mm/teeth	Example 13-3
k	1	Full	page 666
N (pinion (1))	13.84665	teeth	EQ 13-11
Np (1,3)	14	teeth	EQ 13-11
Ng (2,4)	70	teeth	Chapter 18 part 2
d_1,3	24.5	mm	EQ 13-2
d_2,4	122.5	mm	EQ 13-2
(Φ)	20	Deg	Assumption
P	0.571429	teeth/mm	Chapter 18

Angular Velocity, Torque, and Transmitted Loads

In order to find the angular velocity of shaft B we used equation 13-5:

$$\omega_B = \frac{N_p}{N_G}\omega_{input} = \frac{14}{70}(2000\ rpm) = 400\ rpm$$

For the sake of convenience, we also converted all angular velocities to units of radians per second:

$$\omega_A = 2000\ rpm * \frac{1\ min}{60\ s} * 2\pi\ \frac{rad}{rev} = 209\frac{rad}{s}$$

$$\omega_B = 400 rpm * \frac{1\ min}{60\ s} * 2\pi\ \frac{rad}{rev} = 41.9\frac{rad}{s}$$

$$\omega_C = 80\ rpm * \frac{1\ min}{60\ s} * 2\pi\ \frac{rad}{rev} = 8.38\frac{rad}{s}$$

To find the torque on each gear, we used equation 13-33 as, shown below:

$$H = T\omega = \left(W_t \frac{d}{2}\right)\omega \quad (eq. 13-33)$$

$$T_1 = \frac{H}{\omega_A} = \frac{350W}{209\frac{rad}{s}} = 1.67\ Nm$$

$$T_2 = T_3 = \frac{H}{\omega_B} = \frac{350W}{41.9\frac{rad}{s}} = 8.36\ Nm$$

$$T_4 = \frac{H}{\omega_C} = \frac{350W}{8.38\frac{rad}{s}} = 41.8\ Nm$$

Our torque calculations are collected into Table 2.

Table 2: Torque Calculations

Property	Value	Units	Source
$\boldsymbol{\omega_A}$	2000	rpm	Given
$\boldsymbol{\omega_B}$	400	rpm	Chapter 18
$\boldsymbol{\omega_C}$	80	rpm	Given
H	350	W	Given
T1	1.671127	Nm	EQ 18-1
T2	8.355635	Nm	EQ 18-1
T4	41.77817	Nm	EQ 18-1

We then applied equation 13-33 once again to determine the tangential force component transmitted between our gears.

$$T\omega = \left(W_t \frac{d}{2}\right)\omega \rightarrow W_t = 2\frac{T}{d} \quad (eq. 13-33)$$

$$W_{t,1} = \frac{2(1.67\ Nm)}{.0245\ m} = 136.42\ N \ \mid \ W_{t,2} = \frac{2(8.36\ Nm)}{.1225\ m} = 136.42\ N$$

$$W_{t,3} = \frac{2(8.36\ Nm)}{.0245\ m} = 682.09\ N \ \mid \ W_{t,4} = \frac{2(41.8\ Nm)}{.1125} = 682.09\ N$$

After finding the transmitted load, it was a matter of simple trigonometry to find the radial component.

$$W_r = W_t * \tan(\emptyset)$$

$$W_{r,1} = 136.42\ N * \tan(20°) = 49.7N \ \ | \ \ W_{r,2} = 136.42\ N * \tan(20°) = 49.7N$$

$$W_{r,3} = 682.09\ N * \tan(20°) = 248N \ \ | \ \ W_{r,4} = 682.09\ N * \tan(20°) = 248N$$

The results of these calculations are summarized in Tables 3 and 4 below:

Table 3: Transmitted Loads

Property	Value	Units	Adj. Value	Adj. Units	Source
Wt,1	0.13641852	kN	136.4185227	N	EQ 13-33
Wt,2	0.13641852	kN	136.4185227	N	EQ 13-33
Wt,3	0.68209261	kN	682.0926133	N	EQ 13-33
Wt,4	0.68209261	kN	682.0926133	N	EQ 13-33

Table 4: Radial Loads

Property	Value	Units	Adj. Value	Adj. Units	Source
Wr,1	0.04965228	kN	49.65228165	N	Trigonometry
Wr,2	0.04965228	kN	49.65228165	N	Trigonometry
Wr,3	0.24826141	kN	248.2614082	N	Trigonometry
Wr,4	0.24826141	kN	248.2614082	N	Trigonometry

Face width

Shigley (p.920) recommends using a face width 3 to 5 times the circular pitch of the gear. Based on this, we decided to set the face width at 4 times the circular pitch and rounded up to the nearest millimeter.

$$F = 4\left(\frac{\pi}{P}\right) = 4\left(\frac{\pi}{0.5714\ mm^{-1}}\right) = 21.99mm \rightarrow 22mm$$

We rounded our value up to standardize it and make it easier to manufacture, reducing the cost of our assembly.

Center distance

The center distance is the location of the center of each gear with respect to each other. We calculate how far the center of each gear needs to be from the other for optimal load transference without interference using simple circle geometry detailed in example 13-1.

$$\frac{d_p + d_g}{2} = 73.5mm$$

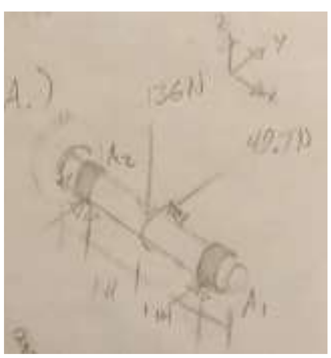
We combined our face width and center distance calculations of our gears as well as the clearances between parts into Table 5.

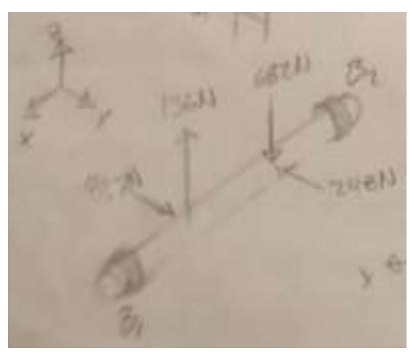
Table 5: Face Width and Clearance

Variable	Value	Units	Citation
F (est.)	21.99115	mm	Chap. 18 p.920
F (standard)	22	mm	Chap. 18 p.920
Center Distance	73.5	mm	Ex 13-1
Clearance	3	mm	Design Choice

Static Analysis

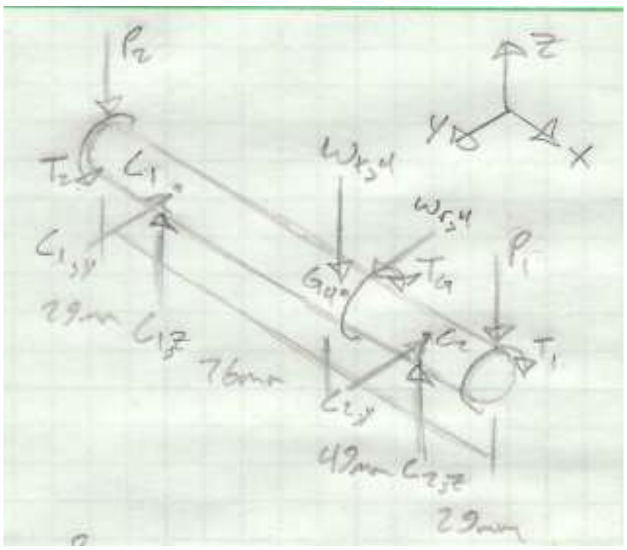
Having determined the contact forces between the gears and having estimated probable loads on the output shaft due to the weight of the rider, we performed static analysis on the shafts to determine the maximum stresses present and reaction forces at the bearings. Shown here are free-body diagrams of shafts A and B.





For the sake of brevity, this report shall not include the full static analysis of all three shafts. However, a walkthrough of the analysis of shaft C is included below, in order to demonstrate our methods as well as to address certain design considerations unique to shaft C. A full summary of the calculated reaction forces may be found in Table 8.

Static Analysis of Shaft C



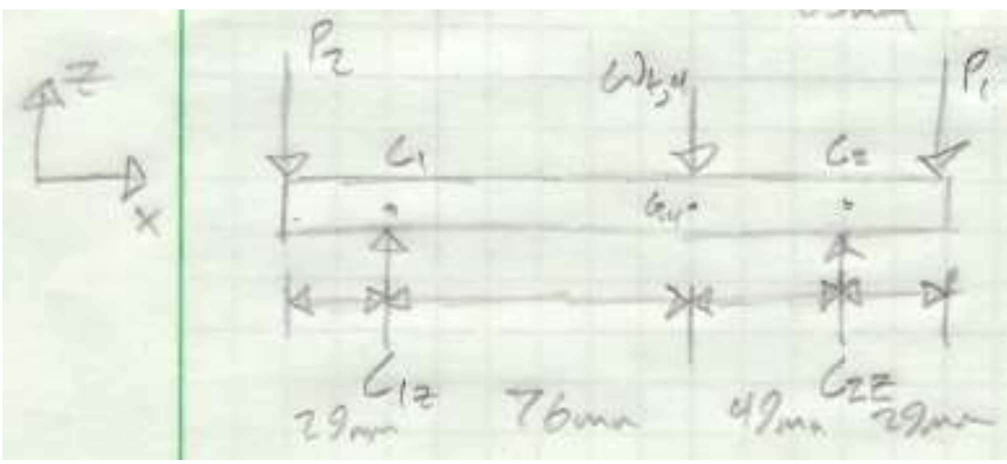
Assuming a rider mass of 120 kg,

$$F_{rider} = mg = 120\ kg * 9.81\frac{m}{s^2} = 1177N$$

For convenience, as well as to ensure our calculations are conservative, we round up to 1200 N. Then,

$$P_1 = P_2 = \frac{F_{rider}}{2} = 600N$$

From Table 3, $W_{t,4} = 682\ N$. Taking the sum of moments about point C_1, we find that



$$\sum M_{C_1} = 0 = 600N * .029m + C_{2,z} * .125m - 682N * .076m - 600N * .154m$$

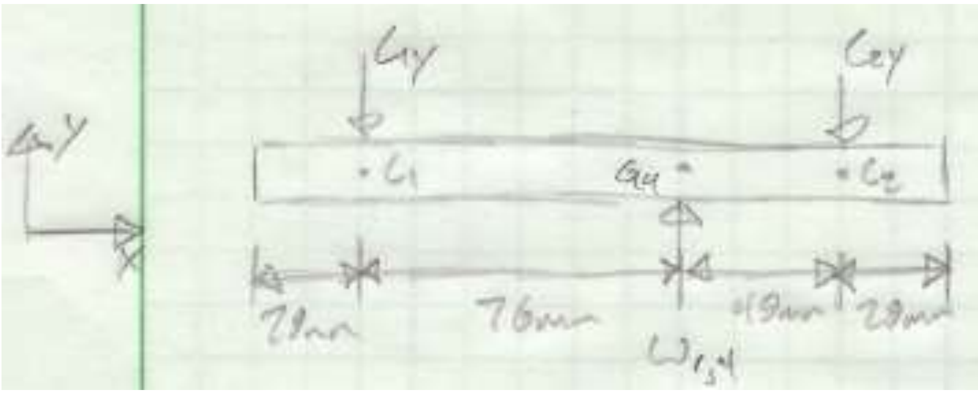
$$C_{2,z} = \frac{1}{.125m} * (682N * .076m + 600N * .154m - 600N * .029m) = \boxed{1015N}$$

Taking the sum of forces in the z-direction allows us to determine $C_{1,z}$:

$$\sum F_z = 0 = 1015N + C_{1,z} - 2 * 600N - 682N$$

$$C_{1,z} = 2 * 600N + 682N - 1015N = \boxed{867N}$$

We apply the same methods to determine the y-components of the bearing reaction forces:



$$\sum M_{C_1} = 0 = C_{2,y} * .125m - 248N * .076m \;\rightarrow\; C_{2,y} = \frac{1}{.125m} * (248N * .076m) = \boxed{151N}$$

$$\sum F_y = 0 = 248N - C_{1,y} - 151N \;\rightarrow\; C_{1,y} = 248N - 151N = \boxed{97N}$$

We compiled our reaction forces into Table 8 below. Then our shaft forces into Table 9:

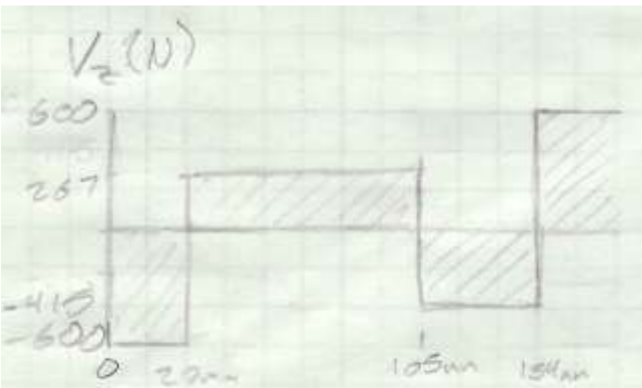
Table 8: Reaction Forces

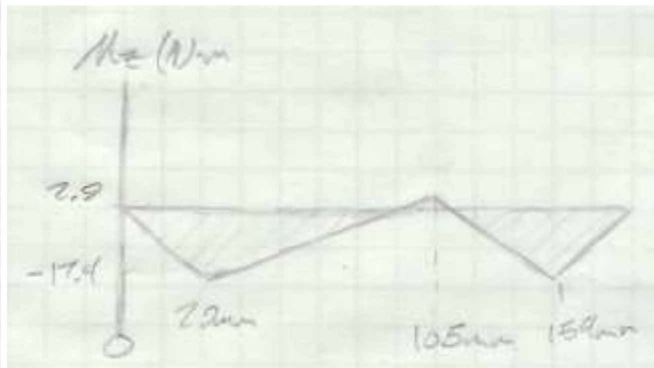
Variable	Value	Units	Adj. Value	Adj. Units	Where it came from
F_A1Z	0.068209	kN	68.20926	N	Statics
F_A1Y	0.024826	kN	24.82614	N	Statics
F_A2Z	0.068209	kN	68.20926	N	Statics
F_A2Y	0.024826	kN	24.82614	N	Statics
FR_A1	0.072587	kN	72.58678	N	Statics
FR_A2	0.072587	kN	72.58678	N	Statics
F_B1Z	0.079792	kN	79.79197	N	Statics
F_B1Y	0.029042	kN	29.0419	N	Statics
F_B2Z	0.465882	kN	465.8821	N	Statics
F_B2Y	0.169567	kN	169.5672	N	Statics
FR_B1	0.084913	kN	84.91284	N	Statics
FR_B2	0.495781	kN	495.7814	N	Statics
F_C1Z	0.867	kN	867	N	Statics

F_C1Y	0.097	kN	97	N	Statics
F_C2Z	1.015	kN	1015	N	Statics
F_C2Y	0.151	kN	151	N	Statics
FR_C1	0.872409	kN	872.4093	N	Statics
FR_C2	1.026171	kN	1026.171	N	Statics

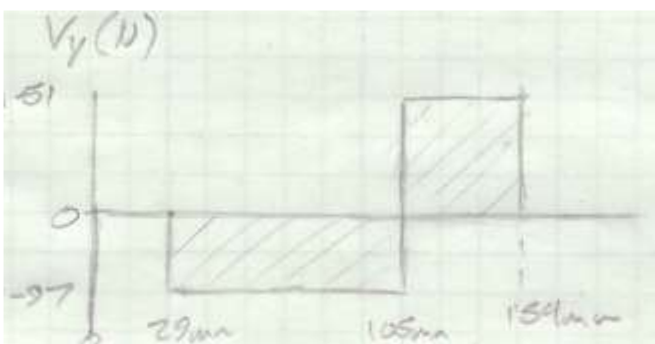
Stress Analysis

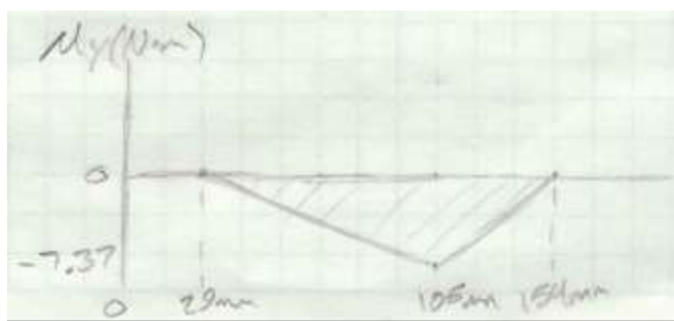
Shear / Moment diagrams along the z-axis:





Shear / Moment diagrams along the y-axis:





After compiling shear force and bending moment diagrams for shaft C, we found the maximum normal and shear stresses in the shaft by analyzing it at the points of maximum loading. The equations we used to calculate these stresses are shown below:

Bending: $\sigma_{bend} = \frac{32M}{\pi d^3}$ Direct Shear: $\tau_{direct} = \frac{4V}{\pi d^2}$ Torsional Shear: $\tau_{tosion} = \frac{16T}{\pi d^3}$

We then calculated the principle stresses in the shaft, and from there the von mises stress:

$$\sigma_1, \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}\,, \quad \sigma' = \left(\frac{(\sigma_1 - \sigma_2)^2}{2}\right)^{1/2}$$

The results of our calculations are summarized below.

Table 9: Maximum Forces on the Shaft

Property	Shaft A	Shaft B	Shaft C	Units
V_max	72.58678	495.7814	618.70914	N
M_max	1.0162149	6.9409396	18.907808	Nm
T_max	1.6711269	8.3556345	143.77817	Nm
Von mises	15.218994	85.295223	314.38621	MPa

Fatigue analysis

We performed a fatigue analysis to make sure our shaft would not suffer a fatigue fracture. DE Goodman was a more conservative analysis to make sure our part would have a longer life span. We performed these calculations on shaft C as it sees the highest stresses, and will therefore fracture first. The calculated values are below in Table 10.

Table 10: Fatigue Analysis

Property	Value	Units	Source
Sut (low) (Chromium alloy high carbon tool steel D3)	2100	MPa	CES Granta
ka (surface finish)	0.35561138	MPa	EQ 6-19 (machined)
kb (size factor)	0.969218478	mm	EQ 6-20
S'e	1050	MPa	EQ 6-18
Kt	2.14		pg 380
Kts	3		pg 380
q	0.55		Fig 6-20
Se	361.8983769	MPa	EQ 6-18
Kf	1.627		EQ 6-32
Ma	1.89078E-05	MNm	
Kfs	2.1		EQ 6-32
Ta	0		
Mm	0		
Tm	0.000143778	MNm	
d (DE Goodman)	0.013719461	m	EQ 6-43

Lewis Equation

The Lewis equation finds the maximum bending stress for the teeth on our gears. We were able to use this equation to identify which gear is likely to fail first and select a material with a higher yield stress than the calculated value for maximum bending stress using a factor or safety of 1.2.

$$\sigma_1 = \frac{W_t P}{FY} = 12.79\ MPa \mid \sigma_2 = \frac{W_t P}{FY} = 8.27\ MPa \mid \sigma_3 = \frac{W_t P}{FY} = 63.96\ MPa$$

$$\sigma_4 = \frac{W_t P}{FY} = 41.35\ MPa$$

The following tables show what values we used for each gear and their source. It is important to note the gear with the largest bending stress was gear 3. We used this bending stress to select our gear material.

Table 11: Lewis Equation for Gear 1

Property	Value	Units	Source
Wt	136.419	N	EQ 13-36
F	0.022	m	Chap. 18 p.920
Kv	1.000		14-6a
P	571.429	teeth/m	EQ 13-2
Y	0.277		Table 14-2
σ	12791834.840	Pa	EQ 14-7
σ	12.792	Mpa	EQ 14-7

Table 12: Lewis Equation for Gear 2

Property	Value	Units	Source
Wt	136.419	N	EQ b on page 686
F	0.022	m	Chap. 18 p.920
Kv	1.000		14-6a
P	571.429	teeth/m	EQ 13-2
Y	0.429		Table 14-2
σ	8269167.446	Pa	EQ 14-7
σ	8.269	Mpa	EQ 14-7

Table 13: Lewis Equation for Gear 3

Property	Value	Units	Source
Wt	682.093	N	EQ b on page 686
F	0.022	m	Chap. 18 p.920
Kv	1.000		14-6a
P	571.429	teeth/m	EQ 13-2
Y	0.277		Table 14-2
σ	63959174.200	Pa	EQ 14-7
σ	63.959	Mpa	EQ 14-7

Table 14: Lewis Equation for Gear 4

Property	Value	Units	Source
Wt	682.093	N	EQ b on page 686
F	0.022	m	Chap. 18 p.920
Kv	1.000		14-6a
P	571.429	teeth/m	EQ 13-2
Y	0.429		Table 14-2
σ	41345837.231	Pa	EQ 14-7
σ	41.346	Mpa	EQ 14-7

Dynamic Load Calculations for Bearings

We used the following three equations to solve for our dynamic load value in order to select a bearing from the Timken catalogue. Ball bearings were deemed appropriate for our design as our reaction forces were not high enough to justify more expensive cylindrical bearings. We assumed L_{10} = 150,000 revolutions to give the bearings a long service life, e = 3 for standard ball bearings, and P_r = F_R on Table 8 because we have standard bearings.

$$L_{10} = \left(\frac{C}{P_r}\right)^e (1 \times (10)^6)$$

We factored in the ABMA expanded life formula in order to account for reliability of our bearing. We have a long service-life, so we set reliability to 90%. Because we are splitting the load between two bearings, we take the square root of 90% to get 95% for our reliability factor. Thus, a1 becomes 0.64.

$$L_{na} = a_1 a_2 a_3 L_{10}; a_2 = a_3 = 0.64$$

This then simplifies into our new dynamic load rating equation which we solve for each of our gears and results in Table 15.

$$C = P_r \left(\frac{L_{na}}{a_1(1 \times (10^6)} \right)^{1/e}$$

Table 15: Bearing Calcs

Property	Value	Units	Source	Adj. Value	Other Units
Pr_A1	72.5867798	N	Timken Manual Pg 41	0.07258678	kN
Pr_B1	84.9128367	N	Timken Manual Pg 41	0.084912837	kN
Pr_B2	495.781402	N		0.495781402	kN
Pr_C1	872	N	Timken Manual Pg 41	0.872	kN
Pr_C2	1026	N	Timken Manual Pg 41	1.026	kN
a1	0.64		Assumption (95% reliability)		
L10	150000	Cycles	Assumption		
e	3		Assumption (Ball bearing)		
C_A1	60.2616903	N	Timken Manual Pg 48	0.06026169	kN
C_B1	70.4948075	N	Timken Manual Pg 48	0.070494807	kN
C_B2	411.598715	N	Timken Manual Pg 48	0.411598715	kN
C_C1	723.93615	N	Timken Manual Pg 48	0.72393615	kN
C_C2	851.787259	N		0.851787259	kN

Gearbox Design and Specifications

Weight of Assembly

Analyzing our design in Solidworks we find our assembly without the motor comes out to roughly 0.98 kg. If an average mountain bike weighs 12.7 kg before the motor and battery, this gear assembly accounts for only a 7.7% mass increase over a standard man-powered mountain bike.

Tolerances

In order to for our bearings to have a snug fit on our shaft we selected a tolerance of k6 for the shaft. This gives our shaft a diameter of 15 mm + 1 to 12 μm and is loose enough a repairman can remove them by hand. The bearing will have a bore diameter of 15 mm with a tolerance of 0 to -8 μm.

Material Selection

For our gears we selected Nylon 6 as the ideal material. This decision was made for several reasons one being the noise reduction that occurs when using Nylon 6 gears as opposed to steel gears. The price per unit volume of Nylon 6 is anywhere from 3.72 - 4.24 $\frac{\$}{kg^3}$ while a 1030 steel has a price per unit volume range of 5.85 - 6.16 $\frac{\$}{kg^3}$. Another deciding factor for using Nylon 6 for our gears is the significant weight reduction over its steel counterpart.

When it came to deciding the materials for the shafts, we ended up having two different materials this came about because shaft C has higher forces exerted on it due to being the shaft in contact with the rider. This prompted us to use AISI D3 steel for shaft C while using a 1030 as rolled steel for the other two shafts. Ultimately this decision was made because the AISI D3 steel has a much higher ultimate strength coming in at $2.1(10^9)$ Pa while 1030 as rolled is $4.95(10^8)$ Pa.

Bearing Selection

For our bearings we partnered with Timken as a trusted supplier of quality bearings. We determined deep groove ball bearings would be suitable as they would be smaller, handle random axial loads, and were lower cost than alternatives. Using the calculated dynamic load ratings, we looked through the deep-groove ball bearing catalogue to find a bearing with a small thickness which had an appropriate allowable dynamic load rating. This led us to bearing number 61702 thin section bearing. This bearing has a 15 mm bore diameter, which fits our 15 mm shaft while being more than strong enough to withstand the radial loads.

Given the ball bearings are sealed within the housing, we don't need any shield to retain grease as the housing will retain the grease for the gears and the rest of the assembly. This bearing works for all bearings in our assembly. This will reduce costs by allowing us to purchase this bearing in bulk and save money on manufacturing by simplifying our design.

Table 16: Bearing Specification

Bearing #	Shield	Bore d (mm)	OD (mm)	Cr (kN)	B (mm)
61702	0	15	21	0.94	4

Shaft Design

Based on the results of our static analysis of shaft C, as well as our fatigue analysis, we concluded that an appropriate shaft diameter would be 15 mm. To ensure uniformity among our gears and bearings, used the same diameter of 15 mm for shafts A and B as well. However, we did select a different material shaft C to account for the high stresses it experienced. Shaft C is made of high-strength AISI D3 steel, while shafts A and B are composed of simple 1030 steel, as rolled. This helps reduce the overall costs of the shafts, while still ensuring a suitable factor of safety.

We chose to use a clearance of 3 mm to prevent interference between the various elements of the gearbox while minimizing unused space along the length of the shaft. The axial positioning of the gears along the length of the shaft will be ensured by plastic spacers between the gears and bearings. The tolerances on the shaft diameter are plus one micrometer minimum and plus twelve micrometers maximum, ensuring a tightness level compliant with the k6 fitting standard from the Timken catalogue. Full specifications of each shaft, including dimensions, tolerances, and materials can be found in the tables below.

Table 17: Shaft A Specifications

Property	Value	Units
Length	32	mm
Diameter	15	mm
tolerance	1, 12	µm
Material	1030 steel, as rolled	N/A

Table 18: Shaft B Specifications

Property	Value	Units
Length	61	mm
Diameter	15	mm
tolerance	1, 12	µm
Material	1030 steel, as rolled	N/A

Table 19: Shaft C Specifications

Property	Value	Units
Length	183	mm
Diameter	15	mm
tolerance	1, 12	µm
Material	AISI D3 steel	N/A

Gear Design

We have two gear sizes, the pinion and the gears. Our gear train has values specified in Table 20. Our teeth numbers have a ratio of 5, 14:70 teeth. This complies with the specifications given to us by eBikes-R-Us to convert the 2000 rpm motor down to a manageable 80 rpm. We standardized our module and face width to reduce the cost of manufacturing the gears, which increases the profits of eBikes-R-Us. We used spur gears for our design because they are a well-researched gear, that is simple to assemble and repair. Our gears are all made from Nylon 6, a thermoplastic, to reduce noise when used.

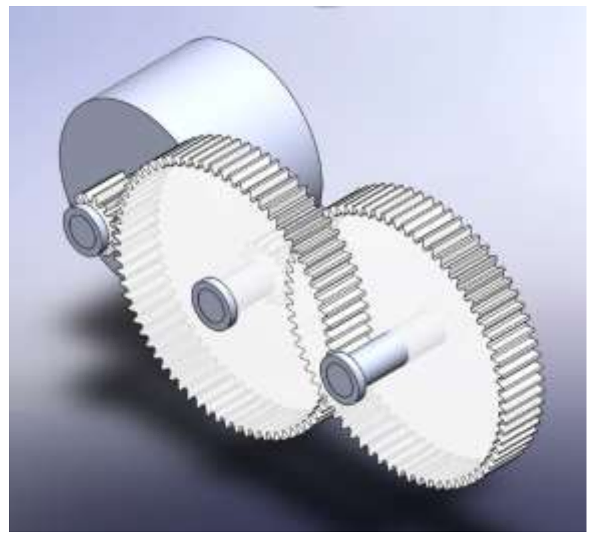


Figure 1: Solidworks model of gear train

Considering the diameter of our gears, we only extend beyond the motor by 32.5 mm. Our entire assembly has a final width of 95 mm, which is well below the requirement of 125 mm specified by eBikes-R-Us.

Finally, we need a ratcheting mechanism between the pedals and shaft C to prevent back pedaling which could damage our motor. This should be a standard part readily available off the shelf for regular mountain bikes. The ratcheting gear could be attached between the pedals and our output shaft while the lever would be mounted on our frame.

Table 20: Gear Specifications

Property	Value	Units
ω (input)	2000	rpm
ω (shaft 2)	400	rpm
ω (output)	80	rpm
Pressure Angle (ϕ)	20	degrees
module	1.75	mm/teeth
N (pinion)	14	teeth
N (gear)	70	teeth
Diameter (gear 1)	24.5	mm
Diameter (gear 2)	122.5	mm
Diameter (gear 3)	24.5	mm
Diameter (gear 4)	122.5	mm
Bore Diameter	15	mm
Diametral Pitch	0.571429	teeth/mm
Face Width	22	mm
Material	Nylon 6	N/A

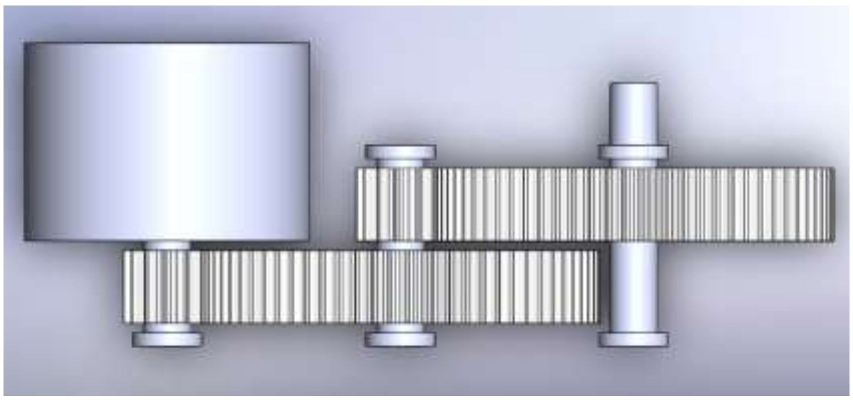


Figure 2: Solidworks model of gear train

Figure 3: Dimensioned diagram of gearbox assembly