Lab 03: Cross Flow Heat Exchanger
October 9, 2020
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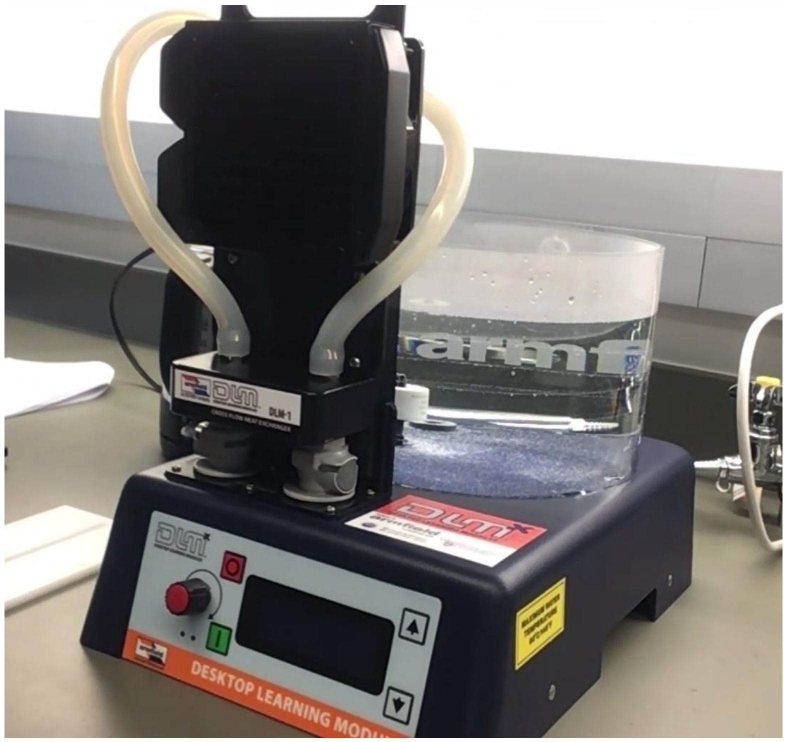
Introduction



Liquid cooling systems are used to exchange heat typically from heat generating surfaces to a gas by using a liquid to transfer the heat to a radiator. Radiators utilize a large surface area by creating multiple layers of many fins attached to an array of pipes. Often radiators include a fan to force air across the fins to further increase heat transfer. In this lab we will test the cooling ability of an extended surface heat exchanger by measuring the temperature in the water before and after it enters the radiator, as well as the air before and after. I expect after a minute of running the air out of the radiator should be warm, heated up by several degrees, and the water from the tank should be cooled by a couple degrees.

Theory

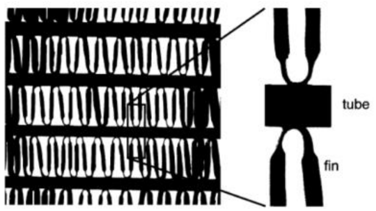


Figure 1: Photograph of a section of the Radiator

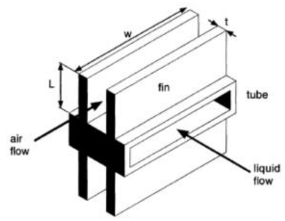


Figure 2: Schematic of a section of flat tube

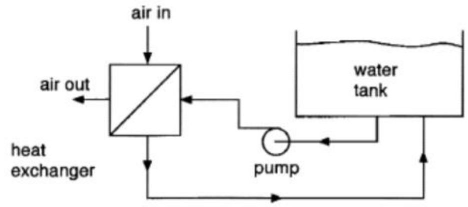


Figure 4: Energy flows in the overall system

Seen in figure 1, fins are connected to the rectangular tube in order to increase surface area to transfer more heat to the surrounding air. In figure 2 we simplify down the rectangular pipe and fins to something more manageable. Figure 4 shows a sketch of our setup, where we have a tank of hot water being pumped through the tubes in the radiator, and air is pushed through the radiator to exchange heat.

In this lab we will use heat equations to calculate the heat transfer from the water to the air, as well as the fin equations in order to calculate the efficiency of the fins.

$$Q_{air} = \dot{m}_{air} \cdot c_{p,air} \cdot (T_{air,out} - T_{air,in})$$

$$Q_{water} = \dot{m}_{water} \cdot c_{p,water} \cdot (T_{water,in} - T_{water,out})$$

The overall heat transfer coefficient for extended area heat exchangers is a function of the inner tube area given below. This is a calculation of the effectiveness of the heat exchanger we are testing.

$$U_i = \frac{1}{\frac{A_i}{[h_o \eta_{o,F}(NA_F + A_b)]} + \frac{x_W}{k_m} + \frac{1}{h_i}}$$

This equation describes the simplified fins in figure 2 through the below variables.

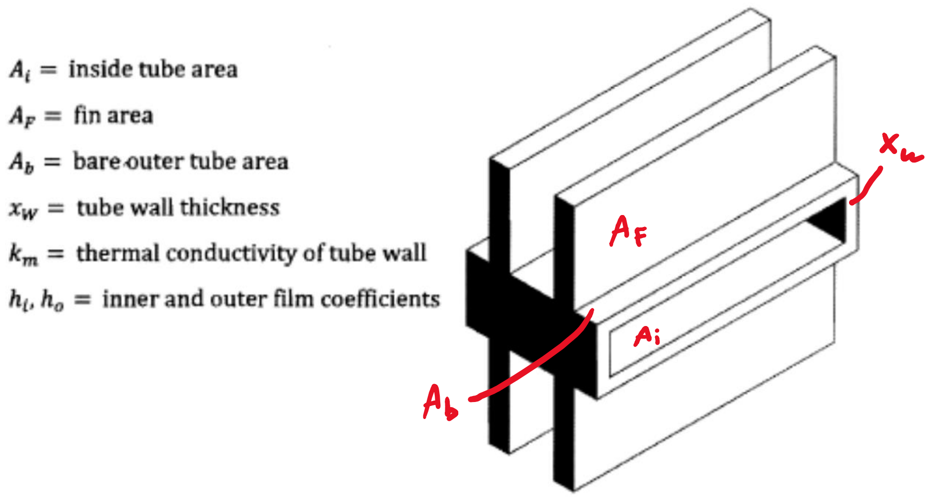


Figure 3: Identify components of a small section of the radiator

Once the overall heat transfer coefficient Ui is calculated, we can find the heat duty of the radiator Qcorrelation

$$Q_{correlation} = U_i A_i F \Delta T_{lm}$$

Where

$$\Delta T_{lm} = \frac{(T_{water,in} - T_{air,in}) - (T_{water,out} - T_{air,out})}{\ln\left(\frac{T_{water,in} - T_{air,in}}{T_{water,out} - T_{air,out}}\right)}$$

And F, the correction factor, is found from the table below for cross-flow heat exchangers.

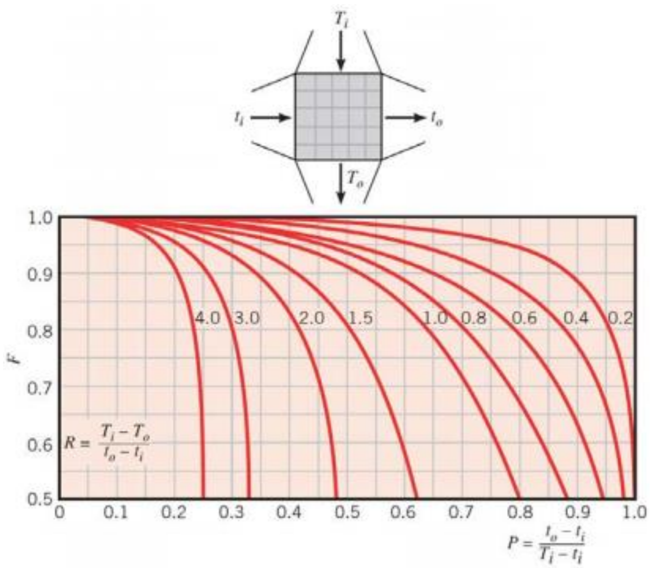


Figure 1: Correction Factor

In order to make simplifications to our calculations, we took extra steps to bring our test unit to equilibrium temperature as well as fully developed flow. These steps included running hot water through the system, and then dumping the water in order to bring the system to not lose any heat to the water tank and tubes. In addition we allowed the machine to run for roughly a minute in order to remove bubbles and fully develop the flow.

Experimental Setup and Procedure

1. Fill and heat two electric kettles with water.
2. Heat up base unit
    a. Fill base unit with two hot kettles and one cold kettle
    b. Wait 1 minute
    c. Dump water from base unit
    d. Refill once more as above
    e. Repeat
    f. Verify water is 40-60 C
3. Install DLM-1 radiator cartridge into base unit
4. Prepare stopwatch, setup thermometers
    a. One thermometer behind radiator, the other in front
5. Measure and record water diameter and depth
6. Turn on base unit, set speed to maximum (4-6 L/min)
7. Wait 1 minute for system to equalize
8. Start timer
9. At times designated in the data table record T in and T out of water and air

Data

Table 1: Temperature Data

Time (s)	t_{in}, water (°C)	t_{out}, water (°C)	T_{in}, air (°C)	T_{out}, air (°C)	Twater avg	Tair avg
0	42.9	42.1	24.1	31.6	315.65	301
30	42.4	41.9	23.8	31.3	315.3	300.7
60	42.1	41.6	23.4	39	315	304.35
90	41.8	41.3	23.5	39.8	314.7	304.8
120	41.6	41	23.3	39.9	314.45	304.75
150	41.4	40.8	23.7	39.9	314.25	304.95
180	41	40.5	23.7	39.7	313.9	304.85

Table 2: Measured Lab Values

Value	Measured Value	Units	Measured Value Base	Units Base
Depth of water in Base Unit	9.7	cm		
Tank Diameter	22.3	cm		
Tank Volume	3788.54	cm^3		
Density of water	997	kg/m^3		
Mass of water in Base Unit	3773.38	g	3.77338	kg
Flow rate	4.55	L/min	0.075605833	kg/s
specific heat of water @ 60s	4179	J/Kgk		
q @ 60s	1187.912381			
density of air	1.18	kg/m^3		
specific heat of air	1007.174	J/Kgk		
kcopper	401	W/mK		
mdot air	0.006	kg/s		
mdot water	0.075605833	kg/s		

Table 3: Tube Data

Value	Variable	measurement	unit	measurement_base	unit base
Tube height	htout	2.1	mm	0.0021	m
Tube width	Wout	13.3	mm	0.0133	m
Tube Length	Lt	110	mm	0.11	m
tall thickness	xw	0.13	mm	0.00013	m
number	N	12	mm	12	(6 per pass)
inner tube width	Win			0.01304	m
inner tube height	hin			0.00184	m
Spacing	S_t	9.3333	mm	0.009333	m

Table 4: Fin Data

Variable	measurement	unit	measurement_base	unit base
xf	16	mm	0.016	m
t	0.11	mm	0.00011	m
w	16.1	mm	0.0161	m
L	3.72	mm	0.00372	m
L_c	3.775		0.003775	

Data Analysis

1. *Determine the heat duty using an energy balance on the air side of the exchanger using the data at a time of 60 sec.*

   Using the Q energy balance equation for air seen below, we can calculate the heat duty on the air side of the heat exchanger. Given values include $\dot{m}_{air} = 0.006\frac{kg}{s}$ and $c_{p,air} = 1005\frac{J}{kg*s}$

$$Q_{air} = \dot{m}_{air} \cdot c_{p,air} \cdot (T_{air,out} - T_{air,in})$$

$$0.006\frac{kg}{s} * 1005\frac{J}{kg*K} * (39.0\text{°C} - 23.4\text{°C}) = \mathbf{94.3\ J}$$

2. *Determine the heat duty using the flow rate and temperature data for the water at 60 sec.*

   We can use a similar equation to calculate Q of the water. $\dot{m}_{water} = 4.55\frac{L}{min} * \frac{1\ min}{60\ s} * 997\frac{kg}{m^3} * 10^{-3} = 0.076\frac{kg}{s}$ was averaged from the flow rate measured by the pump. $c_{p,water} = 4179.0\frac{J}{kg*K}$

$$Q_{water} = \dot{m}_{water} \cdot c_{p,water} \cdot \left(T_{water,in} - T_{water,out}\right)$$

$$= 0.076\frac{kg}{s} * 4179.0\frac{J}{kg*K} * (42.1\text{°C} - 41.6\text{°C}) = \mathbf{157.98\ J}$$

3. *Calculate the fin efficiency factor η_o, from a textbook (Assuming an adiabatic tip).*

$$\eta_{o,F} = \frac{\tanh(mL_c)}{mL_c} \text{ where } m = \sqrt{\frac{hP}{kA_c}} \text{ and } L_c = L + \frac{t}{2}$$

$$h_{0,air} = \frac{Nu_{D,air} * k_{air}}{H_{tube,out}}$$

k for air is a physical constant interpolated for in the textbook tables = 0.0266219

H for the tube is the height of the tube, given to be 2.1 mm

$$N\bar{u}_{D,air} = C_1 Re_{D,max}^{m} Pr^{1/3} * C_2$$

C1 is found to be 0.8 for aligned tube bank in cross flow in the textbook table 7.7.

C2 is 0.977 from a table in the textbook accounting for our few number of tubes (<20).

Pr = Prs as a simplification due to not being able to measure the tube wall temp.

$$Re_{D,max}^{m} = \frac{\rho u L}{\mu}$$

Where rho is the density of air at 315K = 1.18 kg/m^3

L = H tube = 2.1mm

$$u = \frac{\dot{m}_{air}}{\rho_{air} A_{air}} * \left(\frac{S_t}{S_t - H_{tube}}\right)$$

$$A_{air} = A_{total} - N_{tubes} * A_{perp,tube} - N_{fins} * A_{perp,fin}$$
$$= {L_T}^2 - 12L_t * H_{tube} - \frac{2L_t * N_{tubes}}{x_f} * L * t$$

$$A_{air} = (0.112\ m * 0.11\ m) - (12 * 137.5 * (0.00011\ m * 0.00372\ m) + 12 * 0.0021\ m * 0.11\ m) = 0.00887\ m^2$$

St is the spacing between the tubes given in table 3 = 9.333mm

$$u = \frac{0.006 \frac{kg}{s}}{1.18 \frac{kg}{m^3} * 0.00887\ m^2} * \left(\frac{9.333\ mm}{9.333\ mm - 2.1\ mm}\right) = 0.739 \frac{m}{s}$$

$$Re_{D,max}^{m} = \frac{1.18 \frac{kg}{m^3} * 0.739 \frac{m}{s} * 0.0021\ m}{1.867 * 10^{-5}} = 98.168$$

$$N\bar{u}_{D,air} = 0.8 * (98.168^{0.4}) * \left(0.7064^{\frac{1}{3}}\right) * 0.977 = 4.36$$

$$h_{0,air} = \frac{4.36 * 0.0266 \frac{W}{m * K}}{0.0021\ m} = 55.27 \frac{W}{m^2 * K}$$

$$L_c = 3.72\ mm + \frac{0.11\ mm}{2} = 3.775\ mm = 0.003775\ m$$

$$m = \sqrt{\frac{55.27 \frac{W}{m^2 * K} * 0.03242\ m}{410 \frac{W}{m * K} * 1.771 * 10^{-6}\ m^2}} = 49.68\ m^{-1}$$

$$\eta_{o,F} = \frac{\tanh(49.68\ m^{-1} * 0.003775\ m)}{49.68\ m^{-1} * 0.003775\ m} = \mathbf{98.88}\%$$

4. *Calculate h_i and h_o.*

H_o was calculated above as $49.68 \frac{W}{m^2 * K}$. For h_i we use the Sieder-Tate correlation.

$$Nu_{D,water} = 1.86 * \left(Re_{D,w} * Pr_w\right)^{\frac{1}{3}} * \left(\frac{D_h}{L_t}\right)^{\frac{1}{3}}$$

$$D_h = \frac{4A_s}{P} = \frac{4 * (0.00184\ m * 0.01304\ m)}{2 * 0.00184\ m * 0.01304\ m} = 0.00322\ m$$

$$u_w = \frac{\dot{m}_{water}}{\rho_{water} A_s} = \frac{0.0756 \frac{kg}{s}}{997 \frac{kg}{m^3} * 2.399 * 10^{-5}\ m^2} = 3.16 \frac{m}{s}$$

$$Re_w = \frac{\rho_{water} * u_w * D_h}{\mu_w} = \frac{997 \frac{kg}{m^3} * 3.16 \frac{m}{s} * 0.00322\ m}{0.000631 \frac{N * s}{m^2}} = 16104.7$$

$$Nu_{D,water} = 1.86 * (16104.7 * 4.16)^{\frac{1}{3}} * \left(\frac{0.00322\ m}{0.11\ m}\right)^{\frac{1}{3}} = 23.294$$

$$h_i = Nu_{D,w} \frac{k_w}{D_h} = 23.294 * \frac{0.634 \frac{W}{m * K}}{0.00322\ m} = \mathbf{4579.36} \frac{\boldsymbol{W}}{\boldsymbol{m^2 * K}}$$

5. *Calculate $Q_{correlation}$. How does this compare to what you obtained from Q_{air}, and Q_{water}? Why might this measurement not be very accurate?*

$$Q_{correlation} = U_i A_i F \Delta T_{lm}$$

$$\Delta T_{lm} = \frac{(T_{water,in} - T_{air,in}) - (T_{water,out} - T_{air,out})}{ln\left(\frac{(T_{water,in} - T_{air,in})}{(T_{water,out} - T_{air,out})}\right)}$$

$$\Delta T_{lm} = \frac{(42.1 - 23.4) - (41.6 - 39)}{\ln\left(\frac{42.1 - 23.4}{41.6 - 39}\right)} = 8.16\ K$$

$$A_i = (height - 2 \cdot wall\ thickness) \cdot (width - 2 \cdot wall\ thickness)\ \cdot (length\ of\ tube)$$

$$A_i = W_{in} * h_{in} = 0.01304\ m * 0.00184\ m = 0.0393 * 10^{-2}\ m^2$$

$$A_b = (height) \cdot (width) \cdot (length\ of\ tube)$$

$$U_i = \frac{1}{\frac{A_i}{h_o n_{o,F}(NA_F + A_B)} + \frac{x_w}{k_{copper}} + \frac{1}{h_i}}$$

$$= \frac{1}{\frac{0.0393\ m^2}{55.27 \frac{W}{m^2 * K} * 98.88 * (12 * 0.000122\ m^2 + 0.0000279\ m^3)} + \frac{0.00013\ m}{410 \frac{W}{m * K}} + \frac{1}{4579.36 \frac{W}{m^2 * K}}}$$

$$U_i = 301.4 \frac{W}{m^2 * K}$$

F is found from the provided table in Figure 1

$$F = 0.9787$$

Thus we can now calculate Qcorrelation

$$Q_{correlation} = 301.4 \frac{W}{m^2 * K} * 0.0393\ m^2 * 0.9787 * 8.16\ K = \mathbf{94.51\ W}$$

Q correlation is 0.254% different from our Q air value, meaning our Q air value is a more accurate representation of the heat transfer than Q water. This could be a result of the water losing heat in any number of places in the base unit, specifically the large surface area of the water open to free convection with the surroundings.

Q might lose accuracy from a number of factors. At time 0 the flow rate was still increasing which meant the system had less time to achieve and sit at fully developed flow for a while minute. However, throughout the test the flowrate fluctuated, which could also change the behavior of the radiator. Finally, despite heating the system prior to running, the temperature in the system changed rapidly making it difficult to get data exactly at the required time stamps.

$$\%error = \frac{|Q_{correlation} - Q_{air}|}{Q_{correlation}} * 100\% = \frac{|94.51\ W - 94.3\ W}{94.51\ W} * 100\% = \mathbf{0.254\%}$$

Our percent error calculation for Qcorrelation and Qair shows very low error at only 0.254%. Comparatively, Qwater shows a 67.2% error which is disgusting.

Conclusion

Our results showed the radiator managed to transfer heat from the water and into the air at a rate of 98.8% efficiency. Our calculations showed that heat duty of the radiator is 94.51 W. This gave us a 0.254% difference between our Qair and Qcorrelation calculations.

Our results confirmed our theory, and the efficiency was higher than I had expected. Sources of error could have been caused by the system having unsteady flow rates during the procedure, and losing heat to the air from the hot water basin. We could improve the test with insulation on the basin and adding a lid.

Further tables that include calculations at all times of the experiment are available in the appendices.

I learned that cross flow heat exchangers are very efficient even in small sizes. This makes sense given their widespread adoption in power generation and chemical production. I've always thought liquid cooling was unnecessary in my computer, but seeing how effective this small radiator was, I might reconsider it for my next.

References, and Appendices

Sources:

Dr. Collins' notes

T. L. Bergman and A. Lavine, Fundamentals of heat and mass transfer. Hoboken, NJ: John Wiley & Sons, Inc., 2017.

$$Volume\ of\ tank = \pi r^2 * depth\ of\ tank$$

$$Mass\ of\ water = Volume\ of\ the\ tank * \rho_{water}$$

$$height = h_{T,outer} = 2.1\ mm\ |\ width = w_{T,outer} = 13.3\ mm$$

$$Length\ of\ Tube = L_T = 11.0\ cm\ |\ thickness = x_W = 0.13\ mm$$

$$A_F = 2 \cdot L \cdot w + 2 \cdot L \cdot t + w \cdot t$$

$$L = 3.72\ mm\ |\ w = 16.1\ mm\ \ |\ t = 0.11\ mm$$

Table 5: Nusselt Numbers

Nu(air)	Nu(water)	C1 (air)	m (air)	fin eff m
4.375512553	23.28534756	0.8	0.4	49.53339746
4.376939051	23.28986446	0.8	0.4	49.52061634
4.359683505	23.29369165	0.8	0.4	49.67554654
4.35757111	23.29792813	0.8	0.4	49.69456154
4.357805659	23.30141841	0.8	0.4	49.69175981
4.356867704	23.30418479	0.8	0.4	49.70089571
4.357336601	23.30897157	0.8	0.4	49.69667316

Table 6: Reynolds Numbers

Red (air)	Red (water)
99.00712583	16285.89758
99.08293494	16187.83352
98.16840425	16104.71374
98.05682154	16007.30006
98.06920709	15927.0177
98.01968365	15863.3693
98.04443912	15753.20018

Velocities	value	units
Velocity (air)	0.573069865	m/s
u_max	0.739444988	
Velocity (water)	3.16056504	

Table 7: Surface area and Perimeter Values

Areas	Variable	Measurement	Units
Area air passes through		0.00887282	m^2
Area of 12 tubes and their fins		0.00344718	m^2

Inside Cross sectional area of tube	Ai	0.0392832	m^2
	Af	0.016826343	m^2
	Ab	0.002682488	m^3
normal fin area along one tube		0.000056265	m^2
Hydraulic Dameter	D	0.003224946	m
Perimeter	P	0.02976	m
area of the unit	A(unit)	0.0001232	m^2

Table 8: Background Calculations

Ui	fin eff	R	P	F
266.4465933	0.988505821	0.106666667	0.39893617	0.9951
302.1818005	0.98851167	0.066666667	0.403225806	0.9969
303.9212229	0.988440671	0.032051282	0.834224599	0.9787
304.1281532	0.988431943	0.030674847	0.890710383	0.9658
304.0899705	0.988433229	0.036144578	0.907103825	0.9507
304.1878819	0.988429034	0.037037037	0.915254237	0.9437
304.1309279	0.988430973	0.03125	0.924855491	0.9466

Table 9: Log Mean Temperature Calculations

Time	Twater in - Tair (K)	Twater out - Tair out (K)	ln(in/out)	delta T lm
0	18.8	10.5	0.582481613	14.24937684
30	18.6	10.6	0.56230758	14.22708903
60	18.7	2.6	1.973012079	8.160112233
90	18.3	1.5	2.501435952	6.716142377
120	18.3	1.1	2.81159088	6.117533003
150	17.7	0.9	2.978925155	5.639618025
180	17.3	0.8	3.073850053	5.367861059

Table 10: Heat Duty Calculations

Tair out - Tair in	Qair	Twater in - Twater out	Qwater (J)	Qcorr
0	18.8	10.5	0.582481613	14.24937684
30	18.6	10.6	0.56230758	14.22708903
60	18.7	2.6	1.973012079	8.160112233
90	18.3	1.5	2.501435952	6.716142377
120	18.3	1.1	2.81159088	6.117533003
150	17.7	0.9	2.978925155	5.639618025

180	17.3	0.8	3.073850053	5.367861059

Table 11: Convection Coefficient

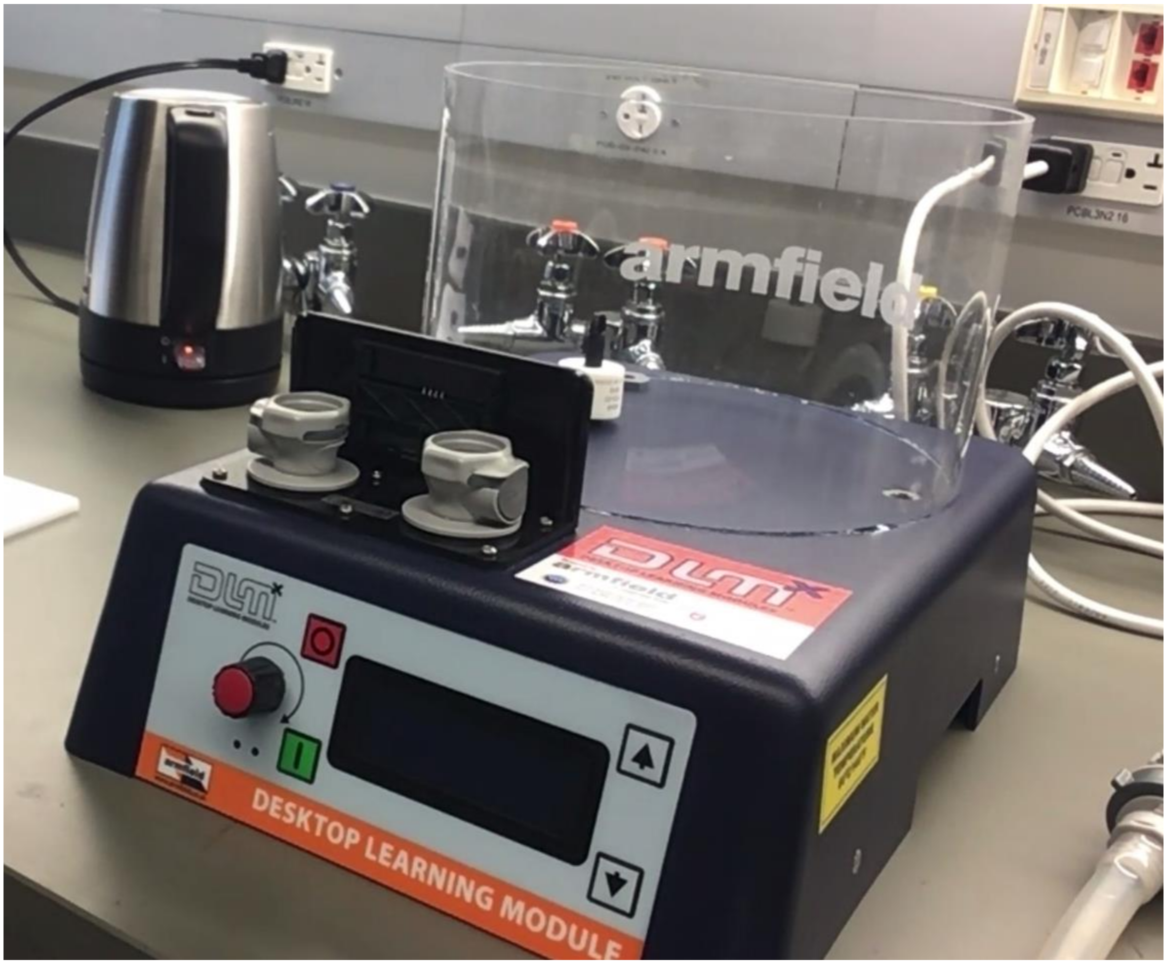
ho (air)	hi (water)
54.95227051	4583.354834
54.92391547	4581.210767
55.268123	4579.363321
55.3104426	4577.595438
55.30420607	4576.113602
55.32454345	4574.922591
55.3151432	4572.826662

Table 12: Physical Constants

Cp_f (J/kg*K)	mu (air)	mu_f (N*s/m^2)	kair	kwater	Pr(water)	P(air)
54.95227051	4583.354834	54.95227051	4583.354834	54.95227051	4583.354834	54.95227051
54.92391547	4581.210767	54.92391547	4581.210767	54.92391547	4581.210767	54.92391547
55.268123	4579.363321	55.268123	4579.363321	55.268123	4579.363321	55.268123
55.3104426	4577.595438	55.3104426	4577.595438	55.3104426	4577.595438	55.3104426
55.30420607	4576.113602	55.30420607	4576.113602	55.30420607	4576.113602	55.30420607
55.32454345	4574.922591	55.32454345	4574.922591	55.32454345	4574.922591	55.32454345
55.3151432	4572.826662	55.3151432	4572.826662	55.3151432	4572.826662	55.3151432

Tabe 13: Misc. Calcualtions

Value	Measured Value	Unit
F	0.9787	@60
N	12	
xw	0.00013	m
Rho	997	kg/m^3


armfield
DESKTOP LEARNING MODULE


Cross Flow
Heat Exchanger
Flow 4.59 l/min
Water In 44.4 degC


field